Some Considerations on Energy Aspect of the Forced Vibrations

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This work reports on a study concerning the behaviour of an elastic system subject to a periodic external excitation. Oscillatory movement is the most common type of mechanical movement found in nature and, of course, in technique. A system executing simple harmonic motion is called harmonic oscillator system. If the harmonic oscillator motion describes a path as the straight line, the harmonic oscillator is called linear harmonic oscillator. Such linear harmonic oscillator is considered an elastic system when the force acts upon it is an elastic one. If in the elastic system dissipative forces are involved then they have the effect of energy losses during oscillation, so that, after a while, the oscillator energy is lost in various forms. To compensate the energy losses of dissipative forces, the oscillator must be driven by a periodic external disturbing force that produces forced oscillations in the elastic systems. If external disturbing force is periodical, the oscillator will run a new type of oscillations called forced oscillations. In this case the period and frequency of forced oscillations of the elastic system will be the same as the period and frequency of disturbing force. In this situation there is a transfer of energy between the two systems. Studying forced vibrations, in terms of energy intake, through excitation forces, are overrun highlight the importance of phase shift between force and oscillator movement.

Keywords: harmonic oscillator system, periodic external excitation, phase shift, energy, resonance phenomenon.

It is known that an elastic system subject to a periodic external excitations makes forced vibrations by the same pulse with external disturbance and with a certain phase difference shift and amplitude depending on the system and excitation characteristics. When their own vibration pulsations is equal with the periodic pulse excitation, or an integer multiple thereof, forced vibrations amplitude increase indefinitely over time, producing a phenomenon known as resonance [1].

The resonance phenomenon can be seen, also, under energy aspect. Elastic system, driven by disturbing forces will be in resonance, unless disturbing force introduces energy into the system; increasing the amplitude of the oscillations can be attributed to elastic system when the excitation energy is transfered to the system.

Experimental part

The interaction of disturbing periodical forces with elastic systems. The elastic system with dumping

We consider, first, the simple case of a linear harmonic oscillator with no damping, free vibration moving, having the own vibrations pulsing described in equation 1, as shown below [2]:

$$x = x \cos \omega t \qquad (1)$$

and an excitation force with ω_1 pulse harmonic variation and φ phase given by the relation number 2:

$$F = F \cos(\omega_1 t + \varphi) \qquad (2)$$

If the force F(t) will interact with the given oscillating system, mechanical work done by force is a measure of the energy transmitted to the system. Depending on this, we can assess whether vibration is amplified or not in time. Mechanical work done by force F(t) or variance of the mechanical energy of the oscillator, over a period of time, will be:

$$W = \int_0^t F(t)dt = \int_0^t F(t)\frac{dx}{dt}dt = -\omega x_0 F_0 \int_0^t \cos(\omega_1 t + \varphi)\sin \omega t dt =$$
$$= \frac{\omega x_0 F_0}{2}\int_0^t \{\sin[(\omega_1 - \omega)t + \varphi] - \sin[(\omega_1 + \omega)t + \varphi]\}dt \qquad (3)$$

For the case when oscillator pulse is the same with the disturbing pulse ($\omega_1 = \omega$ we obtain:

$$W = \frac{\omega x_0 F_0}{2}\left\{t \sin\varphi + \frac{1}{2\omega}[\cos(2\omega t + \varphi) - \cos\varphi]\right\} \qquad (4)$$

Note that, in this case, there is a continuous transfer of energy to the system. This means continuous growth of mechanical energy of the system and, related to this, the continuous growth of oscillation amplitudes. Is the simplest case of resonance of the harmonic oscillator driven by a harmonic perturbation of the same pulsation with it [3]. Increasing energy system in a time $T = 2\pi / \omega$ is obtained from the relation 4 making $t = 2\pi / \omega$ and results the following expression:

$$W = \pi F_0 x_0 \sin\varphi \qquad (5)$$

We observe after equation 5 that the communicated energy is a function of φ phase shift between vibration and disturbing force F. We can remark this in the graph from figure 1 below:

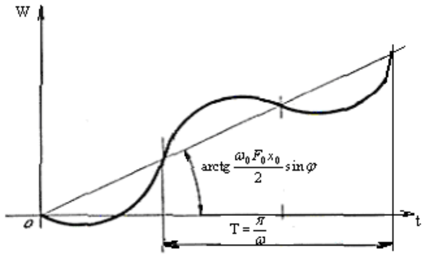

Fig. 1. Harmonic oscillator driven by a harmonic perturbation

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Maximum energy that can be transmitted in a period corresponds of φ = π/ 2 pase shift and is given by formula 6, showing that [4]:

$$W_{\max} = \pi \cdot F_0 x_0 \qquad (6)$$

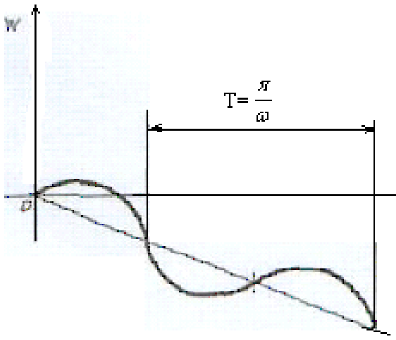

Fig. 2. Harmonic oscillator driven by a harmonic perturbation

For phase shift φ = 0, or φ = π, communicated energy is zero and therefore the force, in phase or in phase opposition with the movement, can not transmit energy to the system although the force pulses and oscillating system pulses are the same. We see, after equation 5 that, in fact, for all negative values of sin φ (π < φ < 2π) W is a negative energy, that energy transfer is from the vibrant system to outside.

As a conclusion, we can communicate mechanical energy to the oscillating system only with forces in advance of moving, with 0 ≤ φ ≤ π angle.

Now, consider the case $\omega_1 = n\omega$ (n = 1, 2, 3 ...), meaning the excitation pulse is an integer multiple of ω pulsation. According to equation 3, corresponding to a time t = 2π/ω, we find:

$$W = \frac{\omega x_0 F_0}{2} \int_0^{\frac{2\pi}{\omega}} \{\sin[(n-1)\omega t + \varphi] - \sin[(n+1)\omega t + \varphi]\} dt = 0$$

Therefore, such an excitation can not continuously transmit energy to the system, that under these conditions resonance can not occur.

Results and discussions

In a general case, when free vibration pulsation is totally different from the ω_1 excitation pulsation, the mechanical work done by force F(t) is obtained by the relation 3 in this form:

$$W = \frac{\omega F_0 x_0}{2(\omega_1 + \omega)} \cos[(\omega_1 + \omega)t] + \varphi - \frac{\omega F_0 x_0}{2(\omega_1 - \omega)} \cos[(\omega_1 - \omega)t] + \varphi + \frac{F_0 x_0 \omega^2}{\omega_1^2 - \omega^2} \cos\varphi \qquad (7)$$

We can put expression 7 in the following form:

$$W = \frac{\omega F_0 x_0}{2} \sqrt{\frac{1}{(\omega_1 + \omega)^2} + \frac{1}{(\omega_1 - \omega)^2} - \frac{2\cos 2\omega t}{\omega_1^2 - \omega^2}} \cos[(\omega_1 + \omega)t + \varphi - \psi] \qquad (8)$$

where:

$$\psi = arctg \frac{\sin 2\omega t}{\cos 2\omega t - \frac{\omega_1 - \omega}{\omega_1 + \omega}}$$

In figure 3 is represented the variation W = W (t) and we can see that the energy exchange oscillates in a quasi-harmonical way around a mean value of the form:

$$W_m = F_0 x_0 \frac{\omega^2}{\omega_1^2 - \omega^2} \cos\varphi \qquad (9)$$

The oscillation of the exchanged energy is harmonic modulated in amplitude, modulation pulsation being the double of the free vibration pulsation.

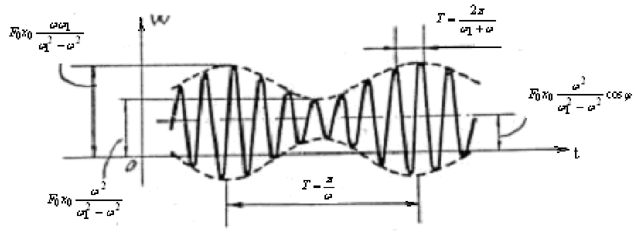

Fig. 3. Quasi-armonic oscillation of energy exchange

The energy accumulated by the oscillator, on account of mechanical work done by the excitation force, periodically varies between two finite limits whose size is a function of the difference between ω and ω_1 pulsations. For $\omega_1 = \omega$, the energy transmitted by the excitation oscillator at a time increases indefinitely, this is the classical resonance case. We admit, further, a periodic force F(t) and a linear harmonic oscillator as before. We assume that the F(t) function satisfies Dirichlet's conditions and can be decomposed in Fourier series as follows below:

$$F(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$

where:

ω- F(t) force pulsation,

A_o, A_i, B_i - coefficients that are determined by Euler's method;

n - integer values of natural numbers.

If the vibrations of the harmonic oscillator are: $x = x_o \cos(\omega_1 t = \varphi)$, then the mechanical work done by exterior force, in a period of time t, is given by the formula:

$$W = \int_0 F(t) \frac{dx}{dt} dt = -\omega_1 x_0 \int_0 F(t) \sin(\omega_1 t + \varphi) dt =$$

$$= -x_0 \omega_1 \int_0 [A_0 + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)] \sin(\omega_1 t + \varphi) dt =$$

$$= x_0 \omega_1 \{ \frac{A_0}{\omega_1} \cos(\omega_1 t + \varphi) + \sum_{n=1}^{\infty} \frac{A_n}{2(n\omega + \omega_1)} \cos[(n\omega + \omega_1)t + \varphi] - \sum_{n=1}^{\infty} \frac{A_n}{2(n\omega - \omega_1)} \cos[(n\omega - \omega_1)t +$$

$$+\varphi] + \sum_{n=1}^{\infty} \frac{B_n}{2(n\omega+\omega_1)} \sin[(n\omega+\omega_1)t+\varphi] - \sum_{n=1}^{\infty} \frac{B_n}{2(n\omega-\omega_1)} \sin[(n\omega-\omega_1)t + \varphi + [\sum_{n=1}^{\infty} \frac{\omega_1 A_n}{(n\omega)^2-\omega^2} +$$

$$\frac{A_0}{\omega_1}]\cos\varphi + \sum_{n=1}^{\infty} \frac{\omega_1 B_n}{(n\omega)^2-\omega_1^2}\sin\varphi \ \}$$

We observe, by the obtained formula, for $\omega_1 = n\omega$, the average value of W energy communicated to the oscillator in a period of time is unlimited. Here we are, in this case, a well-known fact too, namely that the oscillator resonance can occur with each periodic excitation harmonics (the fundamental one and higher ones). For $\omega_1 = n\omega$, if we set aside the terms corresponding harmonics with a pulsation different from pulsation of the free vibration, we obtain:

$$W = -\frac{\omega_1 A_n x_0}{2}\left\{ t\sin\varphi - \frac{1}{2\omega_1}[\cos(2\omega_1 t+\varphi) - \cos\varphi] \right\} -$$

$$-\frac{\omega_1 B_n x_0}{2}\left\{ t\cos\varphi - \frac{1}{2\omega_1}[\sin(2\omega_1 t+\varphi) - \sin\varphi] \right\}$$

W energy, corresponding to this case, contains the first and third term as linear functions of time, marking a continuous transfer of energy from the external to the oscillator.

For $t = 2\pi / \omega_1$, we obtain:

$$W = -\pi \cdot x_0 (A_n \sin\varphi + B_n \cos\varphi)$$

The maximum transmitted energy corresponds to a phase shift of type:

$$\varphi = arctg\frac{A_n}{B_n}.$$

If F(t) is an even function, so that its series development contains only cosine terms, we have for W, corresponding to a period of time, the following expression:

$$W = -\pi \cdot x_0 A_n \sin\varphi$$

For $\varphi = 3\pi/2$ we obtain a maximum of transmitted energy and for $0 < \varphi < \pi$ the energy is not transmitted.

Further consider two particular cases of periodical forces. In the first case we have:

- the force variation embattled shaped, as shown in the graph from figure 4 below:

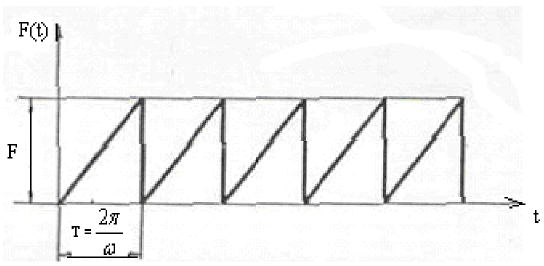

Fig. 4. The F force variation in the form of loophole

In this case, force decomposition in Fourier series is:

$$F(t) = \frac{4F}{\pi}\left(\frac{\sin\omega t}{1} + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + ...\right)$$

In the second case:

- force with a variation in tooth form, as shown in the graph from figure 5 below:

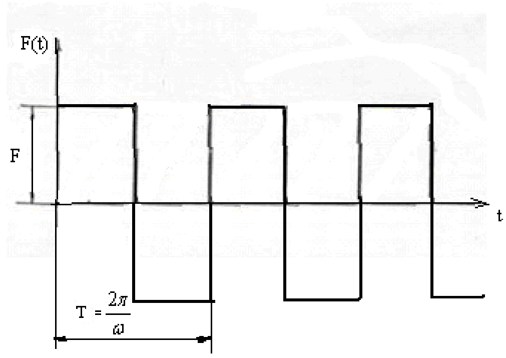

Fig. 5. The F force variation in the form of tooth

In this case, force decomposition in Fourier series in:

$$F(t) = \frac{F}{2} - \frac{F}{\pi}\left(\frac{\sin\omega t}{1} + \frac{\sin 2\omega t}{2} + \frac{\sin 3\omega t}{3} + \frac{\sin 4\omega t}{4} + \frac{\sin 5\omega t}{5} + ...\right)$$

According to those shown earlier, to achieve an energy transfer to a linear harmonic oscillator the first type force, should have its pulsation equal with it or an even and integer sub-multiple of it and the phase shift will be out of and $\pi / 2$ and $3\pi/2$ limits.

For the second case of force (saw teeth) energy transfer is accomplished in the same manner as in the previous case, except that pulsation force must be equal or an integer sub-multiple (odd or even) of the pulse oscillator. Optimal phase shift, for both cases, is $\varphi = 0$. In the sinusoidal shape variation force, the graph will look as in figure 6 below:

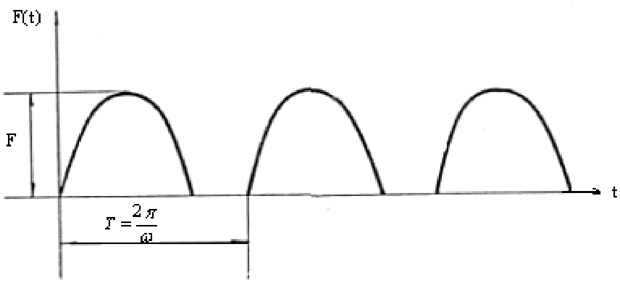

Fig. 6. F variation force in sinusoidal pulse form

In this case, force decomposition in Fourier series is:

$$F(t) = \frac{F}{2} - \frac{F}{\pi}\left(\frac{\sin\omega t}{1} + \frac{\sin 2\omega t}{2} + \frac{\sin 3\omega t}{3} + \frac{\sin 4\omega t}{4} + \frac{\sin 5\omega t}{5} + ...\right)$$

For these forces we have energy transfer to the oscillator in two cases:

- when pulse force coincides with the pulse oscillator and phase shift is out of the $(\pi / 2 .... 3\pi / 2)$ limits;
- when pulse force is an even integer sub-multiple of the pulse oscillator having a phase shift between $(\pi ... 2\pi)$ limits.

Conclusions

In this work we analyzed the interaction between an exciter force and a linear harmonic oscillator assuming the absence of any dissipation and losses of energy. We admited that the mechanical work done by periodic external force is converted entirely into mechanical energy taken by the oscillator. To determine, as before, the external conditions in which energy can be transferred to the oscillator through the excitation force we will have to consider the mechanical work done by the dissipative (damping) forces. Only the difference between the mechanical work done by the external forces and the mechanical work done by the dissipative forces can be considered in this case a measure of the mechanical energy transmitted to the system. Studying force vibrations, in terms of energy intake, through excitation forces, we show the importance of phase shift between force and oscillator movement. We highlight thus unable energy transfer for some phase shift and, also, the optimal phase shift in terms of energy transfer.

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